

B.Sc. Semester – 1 (NEP-2020) Examination

January-2024

Matrices and Coordinate Geometry (Minor)

Time: 2:00 Hours

Marks: 50

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any one.

- (1) Define: Diagonal Matrix, Trace of Matrix, Scalar Matrix, Nilpotent Matrix, Idempotent Matrix. (05)
- (2) Prove that every square matrix can be expressed uniquely as the sum of a symmetric and skew symmetric Matrix. (05)

Que.1(B) Answer any one.

- (1) In usual notation prove that matrix multiplication is associative. (05)
- (2) Let $A=[a_{ij}]$ be $n \times n$ Matrix then prove that $A (\text{adj } A) = |A| I$. (05)

Que.2(A) Answer any one.

- (1) Find rank of matrix $\begin{bmatrix} 3 & 1 & 4 & 0 \\ 0 & 2 & 2 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}$ using definition of linearly independence. (05)
- (2) Prove that the row rank of a matrix is the same as its rank. (05)

Que.2(B) Answer any one.

- (1) Prove that rank of product of two matrices cannot exceed the rank of either matrix. (05)
- (2) Find rank of matrix $\begin{bmatrix} 3 & 1 & 4 & 0 \\ 0 & 2 & 2 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}$ using echelon form. (05)

Que.3(A) Answer any one.

- (1) State and prove Cayley- Hamilton theorem. (05)
- (2) Find eigen values and eigen vectors of Matrix $A = \begin{bmatrix} -3 & -6 & 7 \\ 1 & 1 & -1 \\ -4 & -6 & 8 \end{bmatrix}$. (05)

Que.3(B) Answer any one.

- (1) Solve the linear equations: $2x-y+3z=9$, $x+y+z=6$, $x-y+z=2$. (05)
- (2) Using Cayley- Hamilton theorem find inverse of $\begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$. (05)

Que.4(A) Answer any one.

- (1) In usual notation derive the equation of a conic in polar coordinate $1/r=1+e\cos\theta$. (05)
- (2) Derive formula to find distance between two points in polar coordinates. (05)

Que.4(B) Answer any one.

- (1) Obtain the equation of circle where centre is (p, α) and radius α . (05)
- (2) Find Cartesian coordinates for the polar coordinates $(2, \pi/3)$ and $(-\sqrt{2}, -\frac{\pi}{4})$ (05)

Que.5(A) Answer any one.

(1) In usual notation prove that $\text{adj}(AB) = \text{adj}(B) \text{adj}(A)$ (05)

(2) Find echelon form of matrix $\begin{bmatrix} 1 & 0 & -4 \\ 0 & -1 & 2 \\ -1 & 2 & 1 \end{bmatrix}$. (05)

Que.5(B) Answer any one.

(1) If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ then show that $A^n = \begin{bmatrix} 1 + 2n & -4n \\ n & 1 - 2n \end{bmatrix}$ where $n \in \mathbb{N}$. (05)

(2) Find area of triangle with vertices $(-3, 30^\circ)$, $(5, 150^\circ)$, $(7, 210^\circ)$. (05)
